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# Pre-Service Mathematics Teachers' Conceptual Knowledge of Binary Operation[1]

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## Introduction

Both mathematical knowledge and mathematical knowledge for teaching of teachers or pre-service teachers have been discussed in a substantial body of literature by the researchers (Ball, D. L., Thames, M. H., and Phelps, G. 2008; Fauskanger, J., 2015; Rittle-Johnson, B. and Alibali, M. W., 1999; Siegler, R. S. and Lortie-Forgues, H., 2015). In a study on teacher knowledge Kinach (2002) shows that there is a discrepancy between the objectives of teacher education programs and the knowledge and beliefs of pre-service teachers. Kinach (2002) stated:

> Increasingly teacher educators/researchers report that the subject-matter understanding preservice teachers bring to teacher education coursework is not the sort of conceptual understanding that they will need to develop in their future students (Ball, 1987; Wineburg & Wilson, 1988; Brickhouse, 1990; Thompson, 1992; Ebert, 1993; Magnusson, 1994; Fuller, 1996). In the case of mathematics teacher education, for example, it is well documented in the literature that the procedural understanding of mathematics that preservice

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> teachers typically exhibit in university mathematics courses, mathematics methods courses, and other teacher education coursework is not adequate to teach the reform-mathematics curricula designed to implement the National Council of Teachers of Mathematics (NCTM) Principles and Standards of School Mathematics (Ball, 1988a,b, 1990a, b; McDiarmid, 1990a; Graeber & Tirosh, 1990; Simon, 1993; Kinach, 1996; National Council of Teachers of Mathematics, 2000 (p.52).

A teacher can demonstrate his/her mathematics knowledge in a number of ways. While preparing their course plans, evaluating student work or working with students, they use their mathematics knowledge to establish mathematical relations (Ball et al., 2008; Chick, Pham & Baker, 2006). The mathematical knowledge of an effective teacher consists of types of knowledge such as operational knowledge, conceptual knowledge and mathematical relations (Ball & Bass, 2003).

Skemp (1971) who analyzed mathematical knowledge for the first time with regards to psychological aspect of learning, defines conceptual learning as knowing what to do and why and defines procedural knowledge as the ability to use the rules without understanding their reasons. In other words, while in procedural knowledge there is no need to know the reason for an operation and knowing only how to use an operation is enough, in conceptual knowledge there is an emphasis on understanding (Baki, 1997). Later, Skemp (1976) preferred the terms associative learning instead of conceptual learning by explaining the existence of a piece of knowledge with the associations it has and stated that the number of associations a piece of information forms in itself and with others will play an important role in the understanding of knowledge conceptually (Delice & Sevimli, 2010). Hiebert and Lefevre (1986) define procedural knowledge both as the symbolic language of mathematics and the knowledge of rules and operations used to solve problems. They define conceptual knowledge as a part of the network that include the special parts of information and the relations between those parts. Though these two types of knowledge seem to be independent of each other, procedural knowledge and conceptual knowledge complement each other (Baki 1998). Both conceptual knowledge and operational knowledge are essential to success in mathematics learning (Hiebert & Carpenter, 1992).

In discussing conceptual knowledge of mathematics, Byrnes and Wasik (1991) stated:

> Conceptual knowledge, which consists of the core concepts for a domain and their interrelations (i.e., “knowing that”), has

> been characterized using several different constructs, including semantic nets, hierarchies, and mental models. Procedural knowledge, on the other hand, is "knowing how" or the knowledge of the steps required to attain various goals. Procedures have been characterized using such constructs as skills, strategies, productions, and interiorized actions. (p.777)

After that, Rittle-Johnson and Alibali's (1999) defined conceptual and procedural knowledge as follows:

> We define conceptual knowledge as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. We define procedural knowledge as action sequences for solving problems. (p.175)

It has been shown in literature that mathematics lessons are starting to focus more on procedural learning rather than conceptual learning (Baki 1998), and mathematics courses are carried out with a strong emphasis on conceptual learning and operations are memorized rather than conceptually learned. Schoenfeld (1985) and Hiebert and Lefevre (1986) state that it is not surprising to find students who lack conceptual knowledge or who has very little command of conceptual knowledge in their operations. In fact, some students are not even aware that there are concepts behind the operations they use. Such students cannot understand that there is a meaning in mathematics. Those students believe that mathematics is about carrying out operations on meaningless body of symbols and try to learn mathematical concepts by memorization (Oaks, 1990).

## Purpose of the study

In essence, it is not possible to think of procedural and conceptual knowledge separately. Effective learning in mathematics can only be achieved by balancing procedural and conceptual knowledge. It is believed that this balance can facilitate the higher order mathematical thinking which is necessary for understanding mathematics, logical reasoning, making inferences, drawing generalizations and forming associations between subjects (Birgin & Gürbüz, 2009). This balance should be first established for pre-service teachers.

The main topics of mathematics are sets and functions. The terms in mathematics consisted of sets and functions which are defined on and which have limiting properties on sets. Mathematics is based on analysis and analysis is in turn based on the concepts of functions and operations. Therefore, the learning of operation, which is considered as a type of function, plays an important role in teaching and learning mathematics. As a result of this, it is necessary to determine whether pre-service teachers have a balanced level of conceptual and procedural knowledge on operations (binary operation), one of the mathematics' fundamental topics.

This study focuses on two research questions:

1. What is the conceptual knowledge level of pre-service mathematics teachers regarding binary operation?
2. At what level did conceptual learning about binary operation took place?

## Theoretical Perspective

In order to establish the framework for the study, studies relevant to conceptual knowledge and learning are analyzed. As a consequence of this analysis, a new framework in parallel with these studies is created. An analysis of the literature indicated that researchers used various frameworks to analyze procedural and conceptual knowledge.

Star (2005, 2007) identified two kinds of knowledge, *deep procedural knowledge* and *superficial conceptual knowledge*,

**Table 1**. Types and Qualities of Conceptual and Procedural Knowledge developed by Star

| Knowledge type | Knowledge quality | |
|---|---|---|
| | Superficial | Deep |
| Procedural | Common usage of procedural knowledge | ? |
| Conceptual | ? | Common usage of conceptual knowledge |

**Note:** Reprinted from Star, J. R. (2005:p.408).

Kinach (2002) formed a conceptual learning and procedural learning framework as shown in Table 2.

**Table 2**. Levels of understanding developed by Kinach (cited in Uçar, 2011)

| Procedural | Understanding | Conceptual Understanding |
|---|---|---|
| **Levels of Understanding** | **Subject Level:** Algorithms, terms, rules, knowledge of operations and superficial skills | **Concept Level:** Knowledge and experience on the general thoughts that can direct, define and limit research and exploration in mathematics. |
| | | **Problem Solution Level:** General and topic-based strategies and guiding templates to assess one's line of thinking |
| | | **Epistemological Level:** Proving and justification in a discipline |

**Note:** Reprinted from Z. T. Uçar, (2011:p.89).

The analysis methods used in studies on conceptual and procedural learning/understanding are analyzed. In this study, first of all, student's answers are evaluated as a right or wrong mathematically. Then, right answers were coded based on procedural and conceptual knowledge level. Findings are supported directly by the excerpts from the answers of pre-service teachers. Based on these analyses, the 6 categories given in the table below are created in line with the method of this study and the topic under discussion.

**Table 3.** The framework for the learning level of pre-service teachers

| | **Procedural Learning** | **Conceptual Learning** | **Mis-learning** |
|---|---|---|---|
| **Learning Levels** | **Incomplete Subject: Level:** True but incomplete bookish explanations were given. | **Superficial Conceptual Level:** Answer was given using one's own line of thinking. | Learning did not take place or to learn wrongly. |
| | **Subject Level:** Explanations were given at the rules level based on true bookish knowledge. | **Conceptual Level:** On one's own line of thinking It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information (Hiebert and Lefevre (1986). | |

## Method

### *Research Design*

Qualitative research approach was chosen as the most appropriate method for this study. Qualitative studies allow a deep reflection on the knowledge derived from the data and its meaning (Creswell, 2013). A written test consisting of open ended-questions served as the data collection tool for the study. Open-ended questions help researchers to categorize given answers based on different types of thinking. In this study, two researchers analyzed the answers.

### *Participants*

The participants of the study were primary school pre-service mathematics teachers, high school pre-service mathematics teachers from the faculty of education and 15 pedagogical formation teacher candidates. The participants according to gender and department are given in the table below.

**Table 4**. Participants of the Study

| | Primary School Pre-service Mathematics Teachers | High School Pre-service Mathematics Teachers | Pedagogical Formation Mathematics Teacher Candidates |
|---|---|---|---|
| Male | 12 | 12 | 9 |
| Female | 20 | 18 | 6 |
| Total | 32 | 30 | 15 |

Pedagogical formation students from the Mathematics Department of the Faculty of Sciences were receiving pedagogical formation education at the Faculty of Education. Therefore, they are pre-service teachers. The pre-service teachers from all departments took Introduction to Algebra and Abstract Algebra Courses, which included the subject of binary operation in their undergraduate years.

### *Data Collection and Analysis*

In this study, true-false type questions on binary operations and open-ended questions asking the reason behind true-false statements were being asked. The data was classified based on the whether the true-false choice was true and whether the explanation for the true-false choice was true, false, incomplete or empty. Those who gave the right true-false reply but did not explain it were included in the empty category. Then, the frequencies of data collected from the pre-service teachers' answers were calculated and analyses were carried out using with the help of causal markers, tables and quotations. The data collected was analyzed using descriptive analysis in the context of the framework developed by the researchers.

## Results

This study aimed to determine the conceptual knowledge level of students on binary operations, subjects were first asked the definition of a binary operation. In order for conceptual learning to take place regarding binary operations, it is important to know what binary operations are in the first place. The definition given for binary operation in Abstract Algebra books is as follows:

Definition: Provided that A is a non-empty set $*: AxA \rightarrow A$ transformation is called a binary operation on A (Taşçı, 2007). The analysis of pre-service teachers' definitions of binary operations is given in the table below.

**Table 5**. Frequency tables of teacher candidates' definitions of binary operations

| | True | False | Incomplete | Empty | Total |
|---|---|---|---|---|---|
| Primary School Pre-service Mathematics Teachers | 12 | 15 | 3 | 2 | 32 |
| High School Pre-service Mathematics Teachers | 7 | 13 | 8 | 2 | 30 |
| Pedagogical Formation Mathematics Teacher Candidates | 4 | 6 | 4 | 1 | 15 |

Those who answered incorrectly tried to define binary operation using expressions such as algebraic structure, four operations, arriving at a solution using a certain rule or relations between numbers. It was observed that many candidates have insufficient knowledge about binary operations even at a bookish knowledge level. However, it was also seen that some of those who defined binary operation correctly at a bookish level also understood it properly at a conceptual level.

**Table 6**. Distribution of explanations for binary operations based on level of knowledge

| | False Knowledge | | | True Knowledge | |
|---|---|---|---|---|---|
| | | Incomplete Subject Level | Subject Level | Superficial Conceptual Level | Conceptual Level |
| Primary School Pre-service Mathematics Teachers | 15 | 3 | 6 | 4 | 2 |
| High School Pre-service Mathematics Teachers | 13 | 8 | 5 | 1 | 1 |
| Pedagogical Formation Mathematics Teacher Candidates | 6 | 4 | 1 | 1 | 2 |
| Total | 34 | 15 | 12 | 6 | 5 |

In another questions, subjects were asked to explain whether a given relation was an operation giving reasons in order to determine whether conceptual learning took place. The analysis of the findings is given in the table below.

Upon the analysis of subjects' explanations, it was found that they generally did not pay attention to the set that binary operation defined and while solving the problem they only paid attention to the property of closure but not to the property of well definedness. It was also determined that the subjects who answered these questions wrongly looked at properties such as association, commutative and inverse element.

It is very important to establish relations between topics in conceptual learning. By looking at the following statements, it can be understood that some subjects were not able to learn sets, relations and functions, which needs to be learnt before binary operations.

Pre-service teacher commented that: "If the operations in this relation can be defined using the given sets, these relations can be functions what we talk about here is whether the operations between a and b and aforementioned relations are operations. All of these relations, in fact, carry out an operation."

Pre-service teacher expressed that: "It is not important where " $*,\circ,\triangle,\diamond,\star$ " operations which are defined in all of these options are defined. Since we arbitrarily decide what the operation will be and what it will specify, all of the relations on the side are operations."

**Table 7**. Distribution of data based on the knowledge level of answers on relations

| | False Knowledge | | True Knowledge | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Procedural Level | | | | Conceptual Level | | | |
| | | | Incomplete Subject Level | | Subject Level | | Superficial Conceptual Level | | Conceptual Level | |
| | Number | % | Number | % | Number | % | Number | % | Number | % |
| $In\ N\ a \circ b = a.b$ | 15 | 19 | 35 | 45 | 7 | 10 | 10 | 13 | 10 | 13 |
| $In\ R\ a * b = b/a$ | 30 | 39 | 12 | 15 | 5 | 7 | 15 | 20 | 15 | 20 |
| $In\ Z\ \ a \Delta b = a^b$ | 30 | 39 | 20 | 26 | 2 | 2 | 15 | 20 | 10 | 13 |
| $In\ R\ \ a \circ b = a/(b! - 2)$ | 32 | 40 | 15 | 20 | 5 | 7 | 20 | 26 | 5 | 7 |
| $In\ R^+\ (a/b) * (b/a) = a + b$ | 60 | 78 | 15 | 20 | 2 | 2 | | | | |

The main point where conceptual learning on binary operations did not take place in the subjects was the properties that a relation needs to meet. It is sufficient for a relation to be closed and well defined in its given set for it to be an operation. Commutative property, associative property, identical element and inverse element are the properties that an existing operation can provide on a set.

When subjects were asked, "What properties does a given relation need to satisfy for it to become an operation? "Why?", it was observed that subjects generally looked for other properties. The breakdowns of categories created based on subjects' answers are given in the table below.

**Table 8**. Descriptions of Binary Operation

| Descriptions | Frequency | % (≅) |
|---|---|---|
| It should satisfy closure, association, commutative and identity element properties | 46 | 60 |
| It should be closed and well-defined | 2 | 2 |
| Its set should be defined and closed | 16 | 20 |
| The given relations needs to be a function | 2 | 2 |
| The relation should be a function and should be closed. | 2 | 2 |
| Other | 9 | 11 |
| Total | 77 | 100 |

As can be understood from the table above, 60% of participants stated that for a relation to be an operation it needs to satisfy properties such as closure, association, commutative and identity element. Apart from this, a participant stated that in order to become a binary operation, a relation needs to satisfy the following.

Pre-service teacher commented that: “If a given relation is a bijective function, then it is a binary operation.”

As can be seen, the majority of the participants could not properly explain the properties that a relation needs to satisfy to become an operation. Almost all of them focused on the property of “closure”. There were 3 participants who were able to give a conceptual explanation of the situation correctly. 2 participants emphasized the property of closure and well definedness. A teacher candidate defined binary operation as below without using any of the categories defined above:

Pre-service teacher stated that: “For the given relation to be an operation, ordered pairs need to satisfy two conditions: In a set formed by (x, y) ordered pairs, every element of the domain must form x and that element should have one and only image.”

In other questions assessing whether conceptual learning took place, subjects were given some items that contained judgments. Subjects were asked to state whether these judgments were right or wrong. They were also asked to justify their statements. The frequency table regarding the right and wrong judgments about the items and the completeness or incompleteness of the explanations is given below. As can be seen in the table, a general evaluation of Question 4 shows that

**Table 9.** Frequency table for answers of the true-false test

| Items | False | True | | Total |
|---|---|---|---|---|
| | | Incomplete | True Explanation | |
| If an operation does not provide commutative property, one cannot talk about an identical element. (Wrong Judgment) | 43 | 17 | 17 | 77 |
| There can be no more than one inverse element of any element in an operation. (Right Judgment) | 19 | 19 | 39 | 77 |
| In an operation without an identical element, inverse elements of some elements might exist. (Wrong Judgment) | 17 | 23 | 37 | 77 |
| In an operation, only the inverse of an identical element is equal to itself. (Wrong Judgment) | 33 | 29 | 15 | 77 |

| | | | | |
|---|---|---|---|---|
| In an operation, the inverse of an absorbing element is equal to itself. (Wrong Judgment) | 35 | 33 | 9 | 77 |
| In an operation, if there is no identical element, one cannot talk about an inverse element for this operation. (Right Judgment) | 15 | 29 | 33 | 77 |
| Every element whose inverse is equal to itself is not an identical element. (Right Judgment) | 29 | 37 | 11 | 77 |

It can be seen from the table that most incorrect answers were given for the first item. 43 subjects stated that the given statement was true. In their statement they mistakenly believed that commutative property was one of the main properties of operations and thus they assumed there would be no identical element.

Subject: "In order to find the identical element, the operation should satisfy the commutative property"

Subject: "Because an operation should first meet the commutative property." Let's consider an $*$ operation, and say e is the identical element, $x * e = e * x = x$ should hold.

## Discussion

This study analyzed the conceptual knowledge level of pre-service mathematics teachers regarding binary operation in a written form. In order for pre-service mathematics teachers to teach at a conceptual level in the future, they first need to understand mathematical topics at a conceptual level. The studies on teachers' and pre-service teachers' ability to offer conceptual explanations indicate that their explanations were mostly based on memorization rather than understanding and thus these explanations are rule and operation based (Hiebert & Lefevre, 1986; Kinach, 2002a, 2002b; Kılcan, 2006; Uçar, 2011). Conceptual knowledge and procedural knowledge or conceptual learning and procedural learning are not easy terms to define.

In their study of Star and Stylianides (2013) state that mathematics educators use the same terms (conceptual knowledge and procedural knowledge) in different meanings and that this situation is causing problems in interdisciplinary studies. They recommend two ways to rectify this problem. The first recommendation is to leave conceptual and procedural framework aside and choose a new solution. As a matter of fact, different terms have been in use for centuries. However, this situation might bring about new problems instead of solving them and besides it is not easy to get a new term accepted. It is not clear whether this new term will be able to form a link between the type and quality of information. Secondly, they state that conceptual and procedural knowledge are used differently in mathematics and psychology literatures though they have some similarities. They emphasize the need to clearly define what conceptual and procedural knowledge mean.

Groth and Bergner (2006) analyzed the knowledge structures regarding mean, median and mode in their study. In their study, they highlight that it is not an easy task to teach measures of central tendency at primary school level. They state that in order to achieve this complex conceptual and procedural learning ideas of teacher candidates should be developed. They offer important course design clues to researchers and teacher educators on how they can develop teacher candidates' conceptual and procedural understanding of mean, median and mode.

Uçar (2011) analyzed the instructional explanations of mathematics and form teacher candidates. The results of her study indicate that on certain subjects, mathematics knowledge of pre-service teachers is wrong, their mathematical understanding is generally at a procedural level and accordingly their instructional explanations are at a procedural level. Moreover, the results indicate that pre-service teachers generally deem it enough to give the rules for instructional explanations and do not feel the need to explain why these rules hold. It is noted that teacher candidates with insufficient mathematics knowledge sometimes resort to stylistic tricks as an escape route.

## Implications

This study analyzed the conceptual knowledge level of mathematics teacher candidates regarding binary operations. The analysis of the answers for the questions posed in this study can be taken as an indicator of the fact that concepts are not internalized, and conceptual learning has not fully taken place. This is because in most of the answers of teacher candidates, it is observed that rules about binary operations are directly applied without thinking whether they are appropriate for the questions. There is not a strict division between conceptual and procedural knowledge and their relation with each other cannot be denied. Considering the definitions of these terms, the answers of teacher candidates indicate that their conceptual learning is not at a sufficient level. One of the factors that impact the functionality of the mathematics teaching curricula that have emphasized and prioritized conceptual understanding is the conceptual knowledge competency of mathematics teachers who will implement these curricula. Teacher candidates who will be the teachers of the future need to understand mathematical terms and operations at a conceptual level to teach well. It is not clear how well the teacher candidates who do not have enough conceptual knowledge can ensure the conceptual learning that the curricula specify when they become teachers.

This situation shows that pre-service mathematics teachers have problems in shaping their subject knowledge properly. The functionality of the mathematics teaching curricula that have emphasized and prioritized conceptual understanding in recent years depend on the conceptual knowledge competency of mathematics teachers who will implement these curricula. With regards to Shulman's (1986; 1987) knowledge on teacher competencies, the study indicates that teacher candidates have difficulty in forming their subject knowledge successfully and they need to structure this as soon as possible.

## Limitations and Suggestions

The present study has several limitations, each of which suggests directions for future research. The first limitation was the characteristics of sample and sample size. The current study which limits the generalizability of the results was based on Turkish university students. Thus, using different populations or larger samples could be helpful to improve the generalizability. Second limitation was data collection process. Open-ended questions measures were used to collect the data. Hence, different methods might be used to collect data.

For instance, interviews were also used to investigate pre-service teachers' and students' understanding of the conceptual knowledge. Conceptual knowledge and procedural knowledge may be thought of as separate, they are not broken or independent. Mathematical competence rests on developing both conceptual and procedural knowledge. The courses in the teacher education program are conceptual and can be read out to provide operational information balance.

## Conclusion

This study aims to describe the conceptual knowledge level of pre-service mathematics teachers regarding binary operations. This indicates that pre-service teachers lack some conceptual knowledge underlying operations. The findings of the study indicate that the explanations of teacher candidates were mainly at procedural level and that conceptual learning stood at a superficial level. The majority of the explanations given by pre-service mathematics teachers were based on bookish knowledge or memorization. Few pre-service teachers were able of offer conceptual explanations while many pre-service teachers were not able to explain the underlying meaning and the reasons behind the questions that were posed to them.

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## Summary

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Binary operation is one of the main topics of undergraduate mathematics. A binary operation is also used as a foundation for other disciplines such as physics, chemistry and biology. This study aims to describe the conceptual knowledge level of pre-service mathematics teachers regarding binary operation. In order to achieve this, a test consisting of open-ended and true-false questions on binary operation was administered to a total of 77 pre-service teachers; 32 primary school pre-service mathematics teachers, 30 high school pre-service mathematics teachers and 15 pedagogical formation teacher candidates. The data collected was analyzed using descriptive analysis in the context of the framework developed by the researchers. The findings of the study indicated that the performance of the pre-service teachers was insufficient with regards to the underlying conceptual knowledge that the questions sought. The study also indicates that pre-service teachers have difficulty in forming their subject knowledge successfully and they need to re-structure this as soon as possible.

**Keywords:** Procedural Knowledge, Conceptual Knowledge, Pre-service Mathematics Teachers, Binary Operation

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